the cost associated with the DLS method (without extra computational modes) is approximately double. Two iterations of an IV method could be performed for about the same computer cost as the DLS method (without extra computational DOF), and experience has shown that, in general, IV methods converge in only a few iterations. The cost of oversized LS and DLS methods increases exponentially as the number of computational modes increases. The relative computer cost between an iterative IV method and an oversized LS or DLS method depends upon the application. Reference 3 indicates the advantages of the DLS method over IV and Maximum Likelihood methods for applications with a large number of DOF.

### References

[1]Ibrahim, S. R. and Mikulcik, E. C., "A Method for Direct Identification of Vibration Parameters from the Free Response," *The Shock and Vibration Bulletin,* Vol. 47, No. 4, Sept. 1977, pp. 183-198.

[2]Shinozuka, M., Yun, C. B., and Imai, H., "Identification of Linear Structural Dynamic Systems," *ASCE Journal of Engineering Mechanics Division,* Vol. 108, No. 1, Dec. 1982, pp. 1371-1390.

[3]Ibrahim, S. R., "Double Least Squares Approach for Use in Structural Modal Identification," *AIAA Journal,* Vol. 24, March 1986, pp. 499-503.

[4]Finigan, B. M. and Rowe, I. H., "Strongly Consistent Parameter Estimation by the Introduction of Strong Instrumental Variables," *IEEE Transactions on Automatic Control,* Vol. AC-19, No. 6, Dec. 1974, pp. 825-830.

[5]Ibrahim, S. R. and Pappa, R. S., "Large Modal Survey Testing using the Ibrahim Time Domain Technique," *Journal of Spacecraft and Rockets,* Vol. 19, Sept.-Oct. 1982, pp. 459-465.

[6]Ibrahim, S. R. and Mikulcik, E. C., "A Time Domain Modal Vibration Test Technique," *The Shock and Vibration Bulletin,* Vol. 43, No. 4, June 1973, pp. 21-37.

# Reader's Forum

*Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the* AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

## Comment on "Solutions of One-Dimensional Steady Nozzle Flow Revisited"

George Emanuel*
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REFERENCE 1 discusses a number of topics dealing with one-dimensional nozzle flow. Two of these topics are also discusssed in Refs. 2 and 3 and may well be found in other compressible flow textbooks. These topics are a simple procedure for finding the nozzle solution when there is an internal shock wave, and the criteria for distinguishing different regimes for the flow in a converging/diverging nozzle.

### References

[1]Liou, M.-S., "Solutions of One-Dimensional Steady Nozzle Flow Revisited," *AIAA Journal,* Vol. 26, May 1988, pp. 625-628.

[2]Zucker, R. D., *Fundamentals of Gas Dynamics,* Matrix Pub., Beaverton, OR, 1977, Sec. 6.6.

[3]Emanuel, G., *Gasdynamics: Theory and Applications,* AIAA Education Series, AIAA, Washington, DC, 1986, Sec. 7.1 and App. H.

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## Reply by Author to G. Emanuel

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REGARDING two of many topics discussed in Ref. 1, I wish to thank G. Emanuel for bringing Refs. 2 and 3 to my attention. Indeed, an identical procedure for solving the exit Mach number with a normal shock in a nozzle appears not only in Refs. 1 (Remark 5) and 3 [p. 96, Eq. (7.7)] but also in Ref. 4 [p. 168, Eq. (5.12)]. As demonstrated in Ref. 1 (Remark 1), solution of the total pressure loss across the shock wave is equivalent and straightforward, yielding immediately the shock Mach number. As expressed in Eq. (14) of Ref. 1, the three critical points (called in Refs. 2 and 4) now can be described by the only two solutions of Eq. (13). Consequently, these two solutions completely delineate the seven regimes/points (e.g., see Refs. 2-4) associated with the convergent-divergent nozzle flows.

### References

[1]Liou, M.-S., "Solutions of One-Dimensional Steady Nozzle Flow Revisited," *AIAA Journal,* Vol. 26, May 1988, pp. 625-628.

[2]Zucker, R. D., *Fundamentals of Gas Dynamics,* Matrix Pub., Beaverton, OR, 1977, pp. 155-161.

[3]Emanuel, G., *Gasdynamics: Theory and Applications,* AIAA Education Series, AIAA, Inc., Washington, DC, 1986, pp. 89-101 and 431.

[4]Chapman, A. J. and Walker, W. F., *Introductory Gas Dynamics,* Holt, Rinehart and Winston, Inc., New York, 1971, pp. 149-169.

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## Comment on "Influence of Initial and Boundary Conditions on Vortex Ring Development"

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IRDMUSA and Garris[1] found that vortex rings ejected from a circular hole (air in air) entrained less fluid than

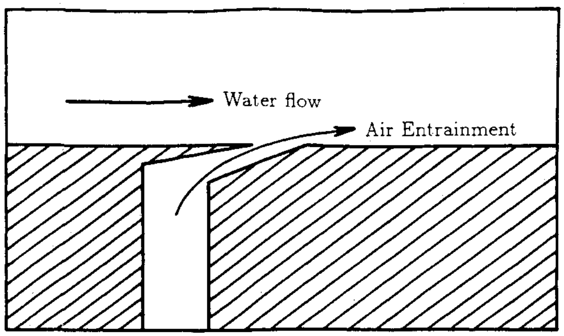


**Fig. 1 A suggestion for optimizing the geometry in canal aeration ducts.**

those ejected from a sharp-edged nozzle. They found also that, all other things being equal, the former moved faster. They suggested that the reason for this lay in the increased detachment of secondary vorticity (with opposite sense) from the leeside wall for the hole case compared to the nozzle case.

It is easy to show[2] that these two effects are explainable in terms of simple inviscid flow considerations; i.e., both effects depend not so much on the detachment of secondary vorticity as on the intrinsic inviscid motion of vortices being generated at a sharp edge at various orientations to the main flow. According to similarity theory predictions using a point vortex model, for example, a growing vortex (i.e., with increasing diameter) should move away from the salient edge at right angles to it, regardless of the orientation of the edge (hole or nozzle) to the main flow. This means that a vortex generated at a hole would not grow at all as it moved in the direction of the issuing jet. On the other hand, a vortex generated at a nozzle would grow and entrain, not moving forward at all. Now this simple model is obviously unrealistic, but it does show both tendencies found by Irdmusa and Garris.[1] The actual growth of the vortex at a hole geometry seems to be accounted for when the effect of the constantly winding-up vorticity is included in the model.[2] The model with this inclusion, however, predicts that a vortex generated at a nozzle would move backward! It would seem that the nozzle geometry is more complicated to calculate, requiring both nonlinear inviscid effects and the inclusion of the notion of viscous entrainment, not easily accountable for using a simple inviscid theory.

Such knowledge seems to have been tacitly used by designers of many forms of entrainment devices. For example, the heart of the water-jet pump is such a sharp-edged nozzle. It is, however, interesting to note that water-jet pumps generally entrain both air and water. As a final point, our experiments[2] show that the motion of a two-dimensional vortex pair obeys essentially the same laws as those of a ring. An example of a planar entrainment device working on the same principle as that of the water-jet pump is the well-known aeration duct nowadays built into canals between bed blocks in order to reduce erosion. These ideas suggest that the form shown in Fig. 1 would probably entrain air more efficiently than the conventional slit form.

A final point on entrainment optimization is that the optimal entrainment for the nozzle geometry found by Irdmusa and Garris[1] can be further improved. Müller and Didden[3] found that the rate of entrainment during the evolution of a single vortex was constant until the "length" of ejected fluid exceeded the nozzle diameter. Rings generated on ejecting a still larger volume of fluid have a reduced rate of entrainment. This finding is in agreement with many engineering applications requiring optimal entrainment that resort to the controlled periodic ejection of small amounts of fluid.

## References

[1]Irdmusa, J. Z. and Garris, C. A., "Influence of Initial and Boundary Conditions on Vortex Ring Development," *AIAA Journal*, Vol. 25, March 1987, pp. 371–372.

[2]Auerbach, D. E., "Experiments on the Trajectory and the Circulation of the Starting Vortex," *Journal of Fluid Mechanics*, Vol. 183, pp. 185–198.

[3]Müller, E.-A. and Didden, N., "Zur Erzeugung der Zirkulation bei der Bildung eines Ringwirbels an einer Düsenmündung," *Strojnícky Časopis*, Vol. 31, 1980, pp. 363–372.

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